### Chapter 3 – Time value of Money

This chapter discusses how to calculate the present value, future value, internal rate of return, and modified internal rate of return of a cash flow stream. Understanding how (and when) to use these formulas is essential to your success as a financial manager! Formulas and examples are included with these notes.

Numbers are rounded to 4 decimal places in tables and formula. However, the actual (non-rounded) numbers are used in the calculations.

#### **Time Value of Money Concepts**

#### 1. Time-Line Conventions

- A. \$1 received today (cash inflow)
- B. \$1 paid in five years (cash outflow)
- C. \$1 received at the end of the third year
- D. \$1 received at the beginning of the third year
- E. Four-year annuity of \$1 per year, first cash flow received at t = 1 (ordinary annuity)
- F. Four-year annuity of \$1 per year, first cash flow received at t = 0 (annuity due)
- G. Four-year annuity of \$1 per year, first cash flow received at t = 2 (deferred annuity)

	0	1	2	3	4	5
А.	\$1					
В.						-\$1
C.				\$1		
D.			\$1			
E.		\$1	\$1	\$1	\$1	
F.	\$1	\$1	\$1	\$1		
G.			\$1	\$1	\$1	\$1

## 2. Notation

 $C_0 = \operatorname{cash} flow at time 0$ 

C = cash flow (used when all cash flows are the same)

r = discount rate or interest rate

t = time period (e.g., t = 4), or number of years (e.g., t years in the future)

m = number of compounding periods per year (e.g., with monthly compounding, m = 12) g = growth rate in cash flow

# 3. Annual Compounding, Single Payments

A. Future value of \$1 as of time 1. Interest rate = 5%.

0	1	2	3	4	5
\$1	\$0	\$0	\$0	\$0	\$0

Formula:  $C_0 (1 + r)^t = \$1(1.05)^1 = \$1.0500$ Financial Calculator: N = 1, I/Y = 5, PV = -1, PMT = 0, FV = Answer

Note on financial calculators – The calculator inputs described above are for a Texas Instruments BAII Plus calculator. (Many other financial calculators require similar inputs.)

Notice that you enter a -1 as the PV and the solution is  $\pm 1.05$ . Here is the intuition: deposit  $\pm 1$  in the bank (negative cash flow), withdraw  $\pm 1.05$  in one year (positive cash flow). If you had entered  $\pm 1$  as the PV, the solution would be  $\pm 1.05$ .

B. Future value of \$1, as of time 5. Interest rate = 5%.

0	1	2	3	4	5
\$1	\$0	\$0	\$0	\$0	\$0

Formula:  $C_0 (1 + r)^t = \$1(1.05)^5 = \$1.2763$ Financial Calculator: N = 5, I/Y = 5, PV = -1, PMT = 0, FV = Answer

C. Present value of \$1, received at time 1. Discount rate = 5%.

0	1	2	3	4	5
\$0	\$1	\$0	\$0	\$0	\$0

Formula:  $C_1 / (1 + r)^t = \frac{1}{(1.05)^1} = \frac{0.9524}{0.9524}$ Financial Calculator: N = 1, I/Y = 5, PV = Answer, PMT = 0, FV = -1

D. Present value of \$1, received at time 5. Discount rate = 5%.

0	1	2	3	4	5
\$0	\$0	\$0	\$0	\$0	\$1

Formula:  $C_5 / (1 + r)^t = \frac{1}{(1.05)^5} = \frac{0.7835}{0.7835}$ Financial Calculator: N = 5, I/Y = 5, PV = Answer, PMT = 0, FV = -1

#### 4. Compounding periods less than one year

A. Future value of 1, as of time 5. Interest rate = 5%, compounded "m" times per year.

0	1	2	3	4	5
\$1	\$0	\$0	\$0	\$0	\$0

General (non-continuous) formula:  $C_0 (1 + r/m)^{tm}$ Continuous compounding formula:  $C_0 e^{rt}$ Note: "e" = 2.718281828

Semi-annual compounding:  $1(1 + (0.05/2))^{(5)(2)} = 1.280085$ Monthly compounding:  $1(1 + (0.05/12))^{(5)(12)} = 1.283359$ Daily compounding:  $1(1 + (0.05/365))^{(5)(365)} = 1.284003$ Continuous compounding:  $1 e^{(5)(0.05)} = 1.284025$ 

Note: Some may use 360 days as the length of one year, other may take into account leap years (366 days every four years). The effects of these changes (from a 365-day year) are extremely small.

Financial Calculator (for semi-annual): N = 10, I/Y = 5/2, PV = -1, PMT = 0, FV = Answer Financial Calculator (for monthly): N = 60, I/Y = 5/12, PV = -1, PMT = 0, FV = Answer Financial Calculator (for daily): N = 1825, I/Y = 5/365, PV = -1, PMT = 0, FV = Answer

B. Present value of 1, received at time 5. Discount rate = 5%, compounded m times per year.

0	1	2	3	4	5
\$0	\$0	\$0	\$0	\$0	\$1

 $\begin{array}{l} \mbox{General (non-continuous) formula: } C_5 \, / \, \left(1 + r/m\right)^{tm} \\ \mbox{Continuous compounding formula: } C_5 \, / \, e^{rt} \end{array}$ 

Semi-annual compounding:  $1 / [1 + (0.05/2)]^{(5)(2)} = 0.781198$ Monthly compounding:  $1 / [1 + (0.05/12)]^{(5)(12)} = 0.779205$ Daily compounding:  $1 / [1 + (0.05/365)]^{(5)(365)} = 0.778814$ Continuous compounding:  $1 / e^{(5)(0.05)} = 0.778801$ 

Financial Calculator (for semi-annual): N = 10, I/Y = 5/2, PV = Answer, PMT = 0, FV = -1Financial Calculator (for monthly): N = 60, I/Y = 5/12, PV = Answer, PMT = 0, FV = -1Financial Calculator (for daily): N = 1825, I/Y = 5/365, PV = Answer, PMT = 0, FV = -1

## 5. Constant Finite Annuities

A. Four-year annuity of \$1 per year, first cash flow received at t = 1. Interest and discount rate = 5%.

0	1	2	3	4	5
\$0	\$1	\$1	\$1	\$1	\$0

Standard formula for the *future value of a finite annuity* = C  $[(1 + r)^{t} - 1] / r$ 

This standard formula for the *future value of a finite annuity* gives a value as of the last period of the annuity (time 4 in this example). The 1.05 is raised to the fourth power because there are 4 payments in the annuity.

Value as of time $4 = \$1 [(1.05^4 - 1) / 0.05] = \$$	\$4.3101
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You can calculate the value of the cash flows at other points in time by multiplying or dividing by 1+r, where r (the interest and discount rate) is 5% in this example.

For instance, assume you want to know the value of the above cash flow stream at t = 6. Time 6 is two years after time 4. To calculate, use the standard formula to determine the value at t = 4, then <u>multiply</u> by  $1.05^2$  to determine the value at t = 6. (Use the second power because you are calculating the value two years after time 4.) The solution is:

Value as of time $6 = \$1 [(1.05^4 - 1) / 0.05] 1.05^2 =$	\$4.7519

As a second example, assume that you want to know the value of the above cash flow stream at t = 1. Time 1 is three years before time 4. To calculate, use the standard formula to determine the value at t = 4, then <u>divide</u> by  $1.05^3$  to determine the value at t = 1. (Use the third power because you are calculating the value three years before time 4.) The solution is:

Value as of time $1 = \$1 [(1.05^4 - 1) / 0.05] / 1.05^3 =$	\$3.7232
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Therefore, "multiply" when you want to determine the value at a later date, "divide" when you want to determine the value at an earlier date.

Some more examples:

Value as of time $0 = \$1 [(1.05^4 - 1) / 0.05] / 1.05^4 =$	\$3.5460
Value as of time $1 = \$1 [(1.05^4 - 1) / 0.05] / 1.05^3 =$	\$3.7232
Value as of time $2 = \$1 [(1.05^4 - 1) / 0.05] / 1.05^2 =$	\$3.9094
Value as of time $3 = \$1 [(1.05^4 - 1) / 0.05] / 1.05^1 =$	\$4.1049
Value as of time $4 = \$1 [(1.05^4 - 1) / 0.05] =$	\$4.3101
Value as of time $5 = \$1 [(1.05^4 - 1) / 0.05] 1.05^1 =$	\$4.5256
Value as of time 6 = $1[(1.05^4 - 1) / 0.05] 1.05^2 =$	\$4.7519
Value as of time 7 = $1 [(1.05^4 - 1) / 0.05] 1.05^3 =$	\$4.9895

Financial Calculator (time 0): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Divide answer by 1.05<sup>4</sup>. Financial Calculator (time 1): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Divide answer by 1.05<sup>3</sup>. Financial Calculator (time 2): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Divide answer by 1.05<sup>2</sup>. Financial Calculator (time 3): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Divide answer by 1.05<sup>1</sup>. Financial Calculator (time 4): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Financial Calculator (time 5): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Multiply answer by 1.05<sup>1</sup>. Financial Calculator (time 6): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Multiply answer by 1.05<sup>2</sup>. Financial Calculator (time 7): N = 4, I/Y = 5, PV = 0, PMT = -1, FV = Answer. Multiply answer by 1.05<sup>2</sup>.

You can also use the formula for the *present value of a finite annuity* to calculate the value of a cash flow stream at different points in time.

Standard formula for the present value of a finite annuity = C {  $[1 - (1 / (1 + r))^{t}] / r$ }

The standard formula gives a value one period before the first payment of the annuity (time 0 in this example). The 1.05 is raised to the fourth power because there are 4 payments in the annuity.

Value as of time $0 = \$1 \{ [1 - (1/1.05)^4] / (0.05) \} = $ $\$3.54$	Value as of time $0 = $	$1 \{ [1 - (1/1.05)^4] / (0.05) \} =$	\$3.5460
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As before, you can calculate the value at other points in time by multiplying or dividing by 1+r, (1.05 in this example).

Two examples:

Value as of time 1 = \$1 { $[1 - (1/1.05)^4] / (0.05)$ } 1.05 <sup>1</sup> =	\$3.7232
Value as of time $6 = \$1 \{ [1 - (1/1.05)^4] / (0.05) \} 1.05^6 =$	\$4.7519

Financial Calculator (time 0): N = 4, I/Y = 5, PV = Answer, PMT = -1, FV = 0Financial Calculator (time 1): N = 4, I/Y = 5, PV = Answer, PMT = -1, FV = 0. Multiply answer by  $1.05^{1}$ . Financial Calculator (time 6): N = 4, I/Y = 5, PV = Answer, PMT = -1, FV = 0. Multiply answer by  $1.05^{6}$ .

B. Three-year annuity of \$1 per year, first cash flow received at t = 0. Interest and discount rate = 5%.

0	1	2	3	4	5
\$1	\$1	\$1	\$0	\$0	\$0

The standard *future value annuity formula* gives a value as of the last year of the annuity (year 2 in this example). This is a three-year annuity. Therefore, 1.05 is raised to the third power in the formula.

Value as of time $2 = \$1 [(1.05^3 - 1) / 0.05] =$	\$3.1525
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The value at other points in time can be calculated by multiplying or dividing by 1.05, raised to the appropriate power.

Value as of time $0 = \$1 [(1.05^3 - 1) / 0.05] / 1.05^2 =$	\$2.8594
Value as of time $4 = \$1 [(1.05^3 - 1) / 0.05] 1.05^2 =$	\$3.4756

The standard *present value annuity formula* gives a value one period before the first payment of the annuity. Therefore, the formula will give you a value at t = -1. You need to multiply by 1 + r to get the value by t = 0.

Value as of time  $0 = \$1 \{ [1 - (1/1.05)^3] / (0.05) \} 1.05^1 =$  \$2.8594

The values at time 2 and 4:

Value as of time $2 = \$1 \{ [1 - (1/1.05)^3] / (0.05) \} 1.05^3 =$	\$3.1525
Value as of time $4 = \$1 \{ [1 - (1/1.05)^3] / (0.05) \} 1.05^5 =$	\$3.4756

C. Five-year annuity of \$1 per year, first cash flow received at t = 3. Interest and discount rate = 5%.

0	1	2	3	4	5	6	7	8
\$0	\$0	\$0	\$1	\$1	\$1	\$1	\$1	\$0

The standard *future value annuity formula* gives the value as of the last year of the annuity (t = 7 in this example). This is a five-year annuity. Therefore, 1.05 is raised to the fifth power.

	Value as of time $7 = \$1 [(1.05^5 - 1) / 0.05] =$	\$5.5256
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Values at different points in time using the future value annuity formula. A couple of examples

Value as of time $0 = \$1 [(1.05^5 - 1) / 0.05] / 1.05^7 =$	\$3.9270
Value as of time 4 = $1 [(1.05^5 - 1) / 0.05] / 1.05^3 =$	\$4.7732
Value as of time 8 = $1 [(1.05^5 - 1) / 0.05] 1.05^1 =$	\$5.8019

The standard *present value annuity formula* gives the value as of the year before the first payment of the annuity (t = 2 in this example).

Value as of time $2 = \$1 \{ [1 - (1/1 \ 05)^5] / (0 \ 05) \} =$	\$4 3295
$V$ and $u_{3}$ of time $2 = 01 ([1 (1/1.05)]) (0.05) = 0$	$\psi_{7.52}$

Values at different points in time using the present value annuity formula. A couple of examples:

Value as of time $0 = \$1 \{ [1 - (1/1.05)^5] / (0.05) \} / 1.05^2 =$	\$3.9270
Value as of time 4 = \$1 { $[1 - (1/1.05)^5] / (0.05)$ } $1.05^2$ =	\$4.7732
Value as of time $8 = \$1 \{ [1 - (1/1.05)^5] / (0.05) \} 1.05^6 =$	\$5.8019

#### 6. Growing Finite Annuities

A. Four-year growing annuity, growing at 10% per year. First cash flow (equal to \$1) received at t = 1. Interest and discount rate = 5%.

0	) 1 2		3	4	5
\$0	\$1	\$1.1	\$1.21	\$1.331	\$0

Standard formula for the *present value of a finite growing annuity* (for when r is not equal to g) =  $C_{\text{first}} [1 - [(1 + g) / (1 + r)]^t] / (r - g)$ . This formula gives the value one period before the first payment (t = 0 in this example).

 $C_{\text{first}}$  is the first cash flow of the annuity. In this above example,  $C_{\text{first}} = C_1 = \$1$ .

*Values at different points in time using the present value growing annuity formula*. Multiply or divide by 1+r (raised to the appropriate power) to determine the value at other points in time.

Value as of time $2 = \$1 \{ [1 - (1.10/1.05)^4] / (0.05 - 0.10) \} 1.05^2$	\$4.5096
Value as of time 4 = \$1 { $[1 - (1.10/1.05)^4] / (0.05 - 0.10)$ } 1.05 <sup>4</sup>	\$4.9719
Value as of time 5 = \$1 { $[1 - (1.10/1.05)^4] / (0.05 - 0.10)$ } 1.05 <sup>5</sup>	\$5.2205

B. Four-year growing annuity, growing at 10% per year. First cash flow (equal to \$1) received at t = 3. Interest and discount rate = 5%.

0	1	2	3	4	5	6	7
\$0	\$0	\$0	\$1	\$1.1	\$1.21	\$1.331	\$0

The standard formula for the *present value of a growing annuity* gives you a value at time 2 (one period before the first payment). This is a 4-year annuity. Therefore, 1.05 is raised to the 4<sup>th</sup> power.

Value as of time  $2 = \$1 \{ [1 - (1.10/1.05)^4] / (0.05 - 0.10) \}$  \$4.0904

Values at different points in time using the present value growing annuity formula. A few examples:

Value as of time 0 = \$1 { $[1 - (1.10/1.05)^4] / (0.05 - 0.10) $ } /1.05 <sup>2</sup>	\$3.7101
Value as of time $5 = \$1 \{ [1 - (1.10/1.05)^4] / (0.05 - 0.10) \} 1.05^3$	\$4.7351

#### 7. Perpetual constant annuities

A. Perpetual constant annuity of  $1 (\cosh \beta \sin \alpha)$  start at time 1, interest and discount rate = 5%)

0	1	2	3	4	5	$\rightarrow$
\$0	\$1	\$1	\$1	\$1	\$1	\$1

The standard formula for the *present value of a perpetual constant annuity* is C/r. The formula gives you the value one period before the first payment.

Value as of time $0 = \$1 / 0.05$	\$20.0000
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Values at different points in time using the present value perpetual constant annuity formula

Value as of time $1 = (\$1 / 0.05) 1.05^{1}$	\$21.0000
Value as of time $4 = (\$1 / 0.05) 1.05^4$	\$24.3101

B. Perpetual constant annuity of  $1 (\cosh \beta \sin \alpha)$  start at time 0, interest and discount rate = 5%)

0	1	2	3	4	5	$\rightarrow$
\$1	\$1	\$1	\$1	\$1	\$1	\$1

The standard formula for the *present value of a perpetual constant annuity* gives you the value one period before the first payment (t = -1 in this example). Therefore, you need to multiply by 1.05 to get the value at t = 0.

Value as of time $0 = (\$1 / 0.05) 1.05$	\$21.0000
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Values at different points in time using the present value perpetual annuity formula. Two examples:

Value as of time $1 = (\$1 / 0.05) 1.05^2$	\$22.0500
Value as of time $4 = (\$1 / 0.05) 1.05^5$	\$25.5256

C. Perpetual constant annuity of \$1 (cash flows start at time 5, interest and discount rate = 5%)

0	1	2	3	4	5	$\rightarrow$
\$0	\$0	\$0	\$0	\$0	\$1	\$1

The standard formula for the *present value of a perpetual constant annuity* gives you the value one period before the first payment (t = 4 in this example).

Value as of time $4 = (\$1 / 0.05)$		\$20.0000
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Values at different points in time using the present value perpetual annuity formula. Some examples:

Value as of time $0 = (\$1 / 0.05) / 1.05^4$	\$16.4540
Value as of time $5 = (\$1 / 0.05) 1.05^{1}$	\$21.0000

D. Perpetual growing annuity, growing at 3% per year (first cash flow, received at time 1, equals \$1, interest and discount rate = 5%)

0	1	2	3	$\rightarrow$
\$0	\$1	\$1.03	\$1.0609	3% more

The standard formula for the *present value of a perpetual growing annuity* is  $C_{first} / (r - g)$ . The formula gives you the value one period before the first payment.

Value as of time $0 = \$1 / (0.05 - 0.03)$	\$50.0000
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*Values at different points in time using the present value perpetual growing annuity formula.* Some examples:

Value as of time $1 = [\$1 / (0.05 - 0.03)] 1.05^{1}$	\$52.5000
Value as of time 4 = $[\$1 / (0.05 - 0.03)] 1.05^4$	\$60.7753

E. Perpetual growing annuity, growing at 3% per year (first cash flow, received at time 11, equals \$1, interest discount rate = 5%)

10	11	12	13	$\rightarrow$
\$0	\$1	\$1.03	\$1.0609	3% more

The standard formula for the *present value of a perpetual growing annuity* gives you the value one period before the first payment (t = 10 in this example).

Value as of time $10 = [\$1 / (0.05 - 0.03)]$	\$50.0000

*Values at different points in time using the present value perpetual growing annuity formula.* A few examples:

Value as of time 0 = $[\$1 / (0.05 - 0.03)] / 1.05^{10}$	\$30.6957
Value as of time 9 = $[\$1 / (0.05 - 0.03)] / 1.05^1$	\$47.6190
Value as of time $11 = [\$1 / (0.05 - 0.03)] 1.05^{1}$	\$52.5000

F. Two growth-rate example: \$1 at time 1, 10% growth rate until time 4, 3% growth rate after time 4 (in perpetuity). Use a 5% discount rate

0	1	2	3	4	5	$\rightarrow$
\$0	\$1	\$1.1	\$1.21	\$1.331	\$1.3709	3% more

Value the first four payments using the finite growing annuity formula. Value the payments starting at time 5 using the perpetual growing annuity formula

The first four payments

Value as of time $0 = \$1$	[1 -	$(1 \ 10/1 \ 05)^4$	1/(0.05 - 0.10)	\$4 0904
	1 4	(1.10/1.00/	1, (0.05 0.10) 1	$\varphi$ 1.0 / 0 1

The payments starting at time 5

Value as of time $0 = 1$	([(\$1))	$(1 \ 1^3)$	(1.03)1/[	(0.05 - 0.03)]	$\frac{1}{105^4}$	\$56 3934
$v$ and $u_{3}$ or time $v =$	$(\psi \tau)$	1.1 /	(1.05/1/1	(0.05 0.05)	1 / 1.05	$\psi_{J} \psi_{J} \psi_{J$

Solution = \$4.0904 + \$56.3934 = \$60.4837

Note:  $[(\$1) (1.1^3) (1.03)] = \$1.3709 =$  the payment at t = 5

G. Two growth-rate example: \$1 at time 0, 10% growth rate until time 12, 3% growth rate after time 12 (in perpetuity). Use a 5% discount rate

0	1	2		11	12	13	$\rightarrow$
\$1	\$1.1	\$1.21	$\rightarrow$	\$2.8531	\$3.1384	\$3.2326	3% more

Value the first thirteen payments (t = 0 to t = 12) using the finite growing annuity formula. Value the payments starting at time 13 using the perpetual growing annuity formula

The first thirteen payments

Value as of time $0 = \$1$	{ [1 -	$(1.10/1.05)^{13}$	3] / (0.05 – 0.1	$(0) \} 1.05$	<sup>1</sup> \$17.4471
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The payments starting at time 13

Value as of time 0 = {[(\$1) (1.1<sup>12</sup>) (1.03)] / [(0.05 - 0.03)]} / 1.05<sup>12</sup> \$90.0011

Solution = 17.4471 + 90.0011 = 107.4482

Note:  $[(\$1) (1.1^{12}) (1.03)] = \$3.2326 =$  the payment at t = 13

H. Two growth-rate example: \$1 at time 3, 10% growth rate until time 20, 3% growth rate after time 20 (in perpetuity). Use a 5% discount rate

0	1	2	3	4		20	21	$\rightarrow$
\$0	\$0	\$0	\$1	\$1.1	$\rightarrow$	\$5.0545	\$5.2061	3% more

Value the first eighteen payments (t = 3 to t = 20) using the finite growing annuity formula. Value the payments starting at time 21 using the perpetual growing annuity formula

The first eighteen payments

Value as of time  $0 = \$1 \{ [1 - (1.10/1.05)^{18}] / (0.05 - 0.10) \} / 1.05^{2} \}$ 

The payments starting at time 21

Value as of time 0 = { [(\$1) (1.1<sup>17</sup>) (1.03)] / [(0.05 - 0.03)] } / 1.05<sup>20</sup> \$98.1063

Solution = \$23.7689 + \$98.1063 = \$121.8752

Note:  $[(\$1) (1.1^{17}) (1.03)] = \$5.2061$  = the payment at t = 21

# 8. Internal rate of return (IRR)

Definition: the IRR = the discount rate that causes the sum of the present values of all cash flows to equal zero.

IRR calculation examples

A. Two cash flows

0	1	2	3	4	5
-\$1	\$0	\$0	\$0	\$0	\$2

 $-\$1 + \$2 / (1 + r)^5 = \$0$ r =  $(\$2 / \$1)^{(1/5)} - 1 = 14.8698\% = IRR$ Financial Calculator: N = 5, I/Y = Answer, PV = -1, PMT = 0, FV = 2

0	1	2	3	4	5
\$1	\$0	\$0	\$0	\$0	-\$2

 $1 + -2 / (1 + r)^{5} = 0$ r =  $(2 / 1)^{(1/5)} - 1 = 14.8698\% = IRR$ Financial Calculator: N = 5, I/Y = Answer, PV = 1, PMT = 0, FV = -2

B. Perpetual constant annuities

0	1	2	3	4	$\rightarrow$
-\$10	\$1	\$1	\$1	\$1	\$1

 $\begin{array}{l} -\$10 + \$1 \ / \ r = \$0 \\ r = \$1 \ / \ \$10 = 10\% = IRR \end{array}$ 

C. Perpetual growing annuities

0	1	2	3	4	$\rightarrow$
-\$10	\$1	\$1.03	\$1.0609	\$1.0927	3% more

 $\label{eq:rescaled} \begin{array}{l} -\$10 + \$1 \ / \ (r - 3\%) = \$0 \\ r = (\$1 \ / \ \$10) + 3\% = 13\% = IRR \end{array}$ 

D. Other cash flow patterns - solve by your calculator or computer. Example - finite annuity

0	1	2	3	4	5
-\$3	\$1	\$1	\$1	\$1	\$1

 $\label{eq:response} \begin{array}{l} -\$3 + \$1 \ \{ \ [1 - (1/(1+r))^5] \ / \ r \ \} = \$0 \\ r = 19.8577\% = IRR \\ \mbox{Financial Calculator: } N = 5, \ I/Y = Answer, \ PV = -3, \ PMT = 1, \ FV = 0 \end{array}$ 

# 9. Modified Internal Rate of Return (MIRR)

- Step one: Using the discount rate, take a PV (to time zero) of the negative cash flows
- *Step two*: Using the interest rate, take a FV (to time t, where t is the time of the last cash flow) of the positive cash flows
- Step three: Calculate the IRR of the two cash flows calculated in the first two steps

A. Using an interest and discount rate = 5%, what is the MIRR of the following cash flow stream?

0	1	2	3	4	5
-\$3	\$1	\$1	\$1	\$1	\$1

*Step 1*: PV of negative cash flows (at time 0) = -\$3 *Step 2*: FV of positive cash flows (at time 5) = \$5.5256 *Step 3*: IRR =  $($5.5256 / $3)^{(1/5)} - 1 = 12.9932\% = MIRR$ 

B. Using an interest and discount rate = 5%, what is the MIRR of the following cash flow stream?

0	1	2	3	4	5
+\$3	-\$1	-\$1	-\$1	-\$1	-\$1

*Step 1*: PV of negative cash flows (at time 0) = -\$4.3295 *Step 2*: FV of positive cash flows (at time 5) = \$3.8288 *Step 3*: IRR =  $($3.8288 / $4.3295)^{(1/5)} - 1 = -2.4277\% = MIRR$ 

C. Using an interest and discount rate = 5%, what is the MIRR of the following cash flow stream?

0	1	2	3	4	5
+\$3	\$0	\$0	-\$1	-\$1	\$1

*Step 1*: PV of negative cash flows (at time 0) = -\$1.6865*Step 2*: FV of positive cash flows (at time 5) = \$4.8288*Step 3*: IRR = (\$4.8288 / \$1.6865)<sup>(1/5)</sup> - 1 = 23.4154% = MIRR

# 10. Application – loan amortization schedules

A 30-year home loan has an annual interest rate of 8%. Interest is compounded monthly. What is the monthly payment on a fully amortizing, level payment loan for \$100,000? \$733.7646

Use this payment to filling in the following loan amortization table for the home loan described above.

Month	Beginning	Total Payment	Interest	Principal	Ending
	Balance		Payment	Payment	Balance
0					\$100,000.000
1	\$100,000.000	\$733.7646	\$666.6667	\$67.0979	\$99,932.9021
2	\$99,932.9021	\$733.7646	\$666.2193	\$67.5452	\$99,865.3569
3	\$99,865.3569	\$733.7646	\$665.7690	\$67.9955	\$99,797.3613
4	\$99,797.3613	\$733.7646	\$665.3157	\$68.4488	\$99,728.9125
5	\$99,728.9125	\$733.7646	\$664.8594	\$68.9052	\$99,660.0073

The loan balance will be \$0 after the 360<sup>th</sup> payment.