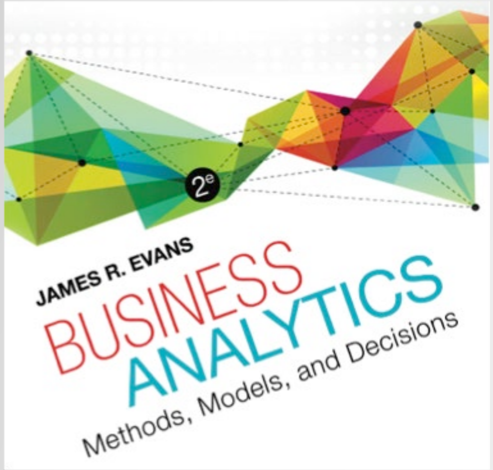


Chapter 9 Forecasting Techniques

Forecasting Techniques

- Managers require good forecasts of future events.
- Business Analysts may choose from a wide range of forecasting techniques to support decision making.
- Three major categories of forecasting approaches:
  1. Qualitative and judgmental techniques
  2. Statistical time-series models
  3. Explanatory/causal models

Qualitative and Judgmental Forecasting

- Qualitative and Judgmental techniques rely on experience and intuition.
- They are necessary when historical data is not available or when predictions are needed far into the future.
- The **historical analogy** approach obtains a forecast through comparative analysis with prior situations.
- The **Delphi method** questions an anonymous panel of experts 2-3 times in order to reach a convergence of opinion on the forecasted variable.

Example 9.1: Predicting the Price of Oil

- Early 1988 – oil price was about $22 a barrel
- Mid-1988 – oil price dropped to $11 a barrel because of oversupply, high production in non-OPEC regions, and lower than normal demand
- In the past, OPEC would raise the price of oil.
- Historical analogy would forecast a higher price.
- However, the price continued to drop even though OPEC agreed to cut production.
- Historical analogies cannot always account for current realities!

Indicators and Indexes

- **Indicators** are measures that are believed to influence the behavior of a variable we wish to forecast.
- Indicators are often combined quantitatively into an **index**, a single measure that weights multiple indicators, thus providing a measure of overall expectation.
  - Example: Dow Jones Industrial Average

Example 9.2: Economic Indicators

- GDP (Gross Domestic Product) measures the value of all goods and services produced.
  - GDP rises and falls in a cyclic fashion.
- Forecasting GDP is often done using **leading indicators** (series that change before the GDP changes) and **lagging indicators** (series that follow changes in the GDP) indicators.
- Examples

Leading - formation of business enterprises
- percent change in money supply (M1)

Lagging - business investment expenditures
- prime rate
- inventories on hand

Example 9.3: Leading Economic Indicators

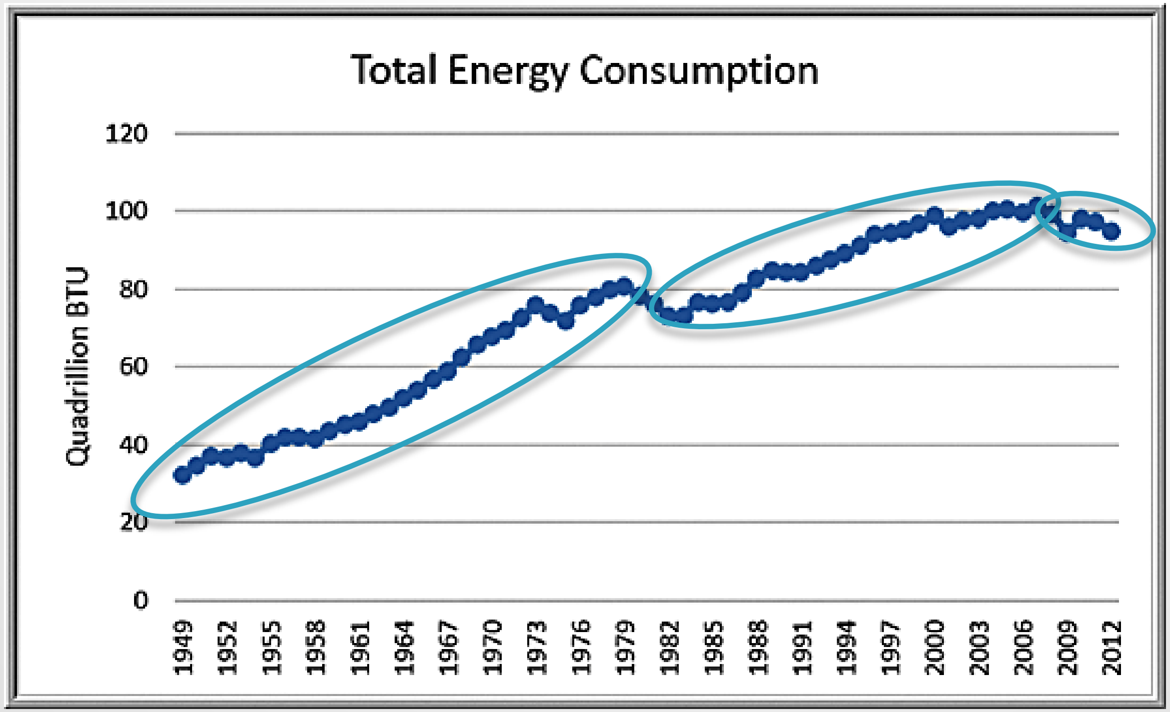
- An *Index of Leading Indicators* was developed by the Department of Commerce.
- This index is related to the economic performance is available from www.conference-board.org.
- It includes measures such as:
  - average weekly manufacturing hours
  - new orders for consumer goods
  - building permits for private housing
  - S&P 500 stock prices

Statistical Forecasting Models

- **Time Series** – a stream of historical data, such as weekly sales
  - *T* = number of periods, *t* = 1, 2, …, *T*
- Time series generally have components such as:
  - random behavior
  - trends (upward or downward)
  - seasonal effects
  - cyclical effects
- **Stationary time series** have only random behavior.
- A **trend** is a gradual upward or downward movement of a time series.

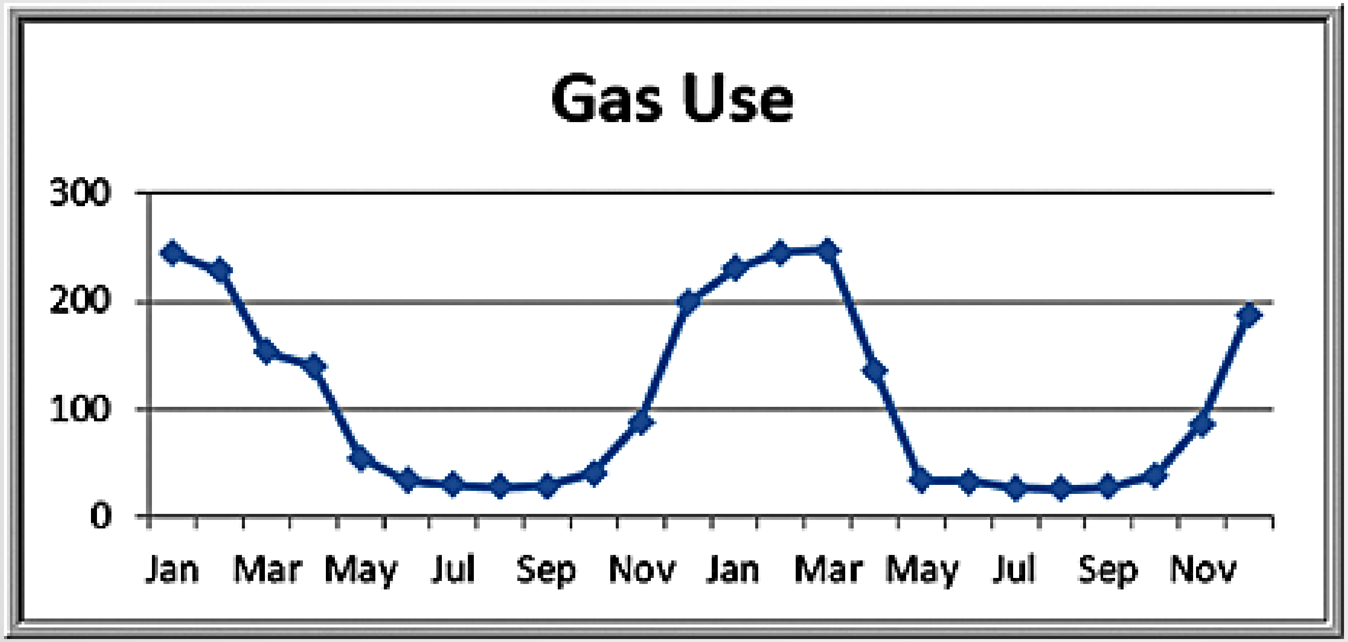
Example 9.4: Identifying Trends in a Time Series

- The *Energy Production & Consumption*
  - General upward trend with some short downward trends; the time series is composed of several different short trends.



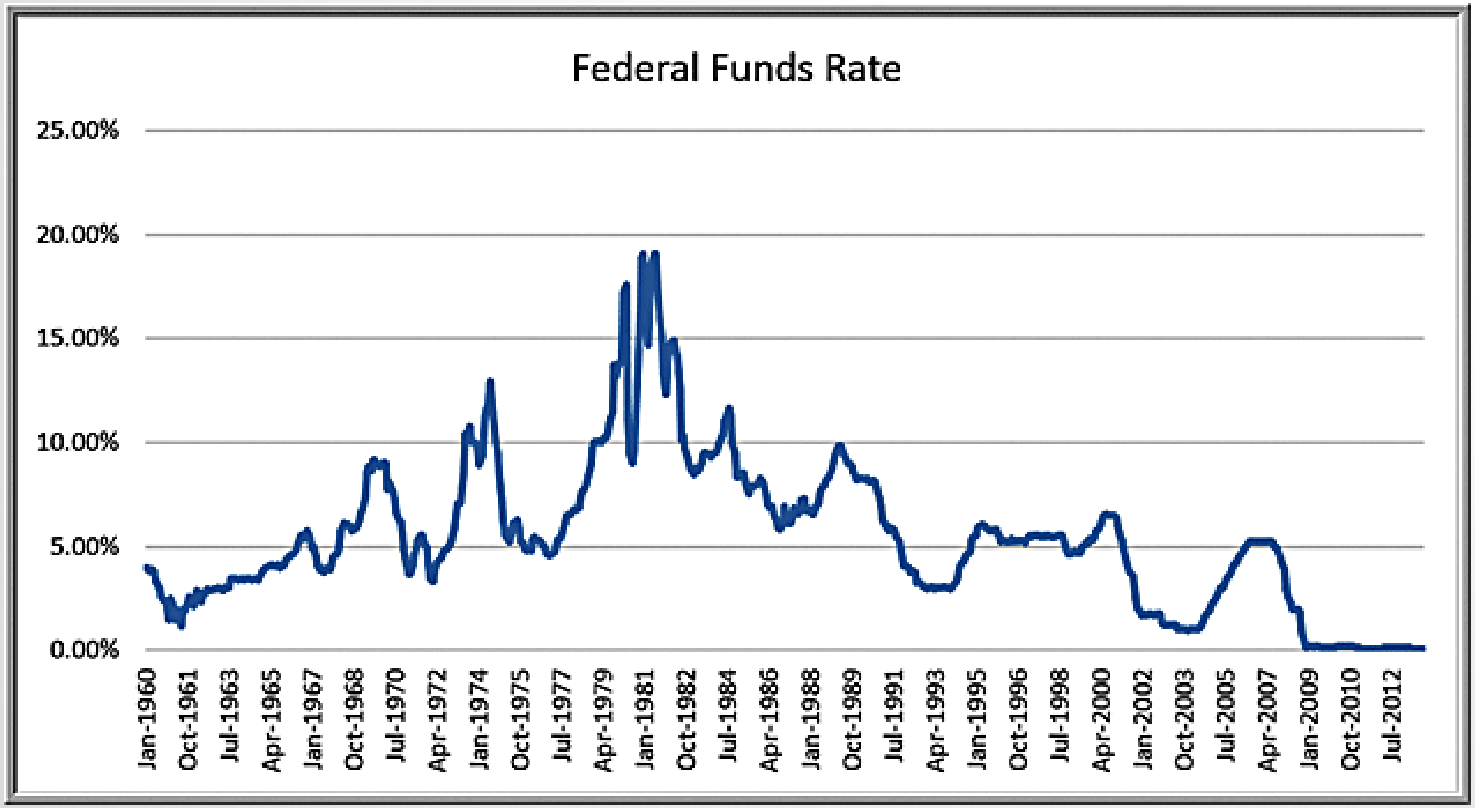
Seasonal Effects

- A **seasonal effect** is one that repeats at fixed intervals of time, typically a year, month, week, or day.



Cyclical Effects

- **Cyclical effects** describe ups and downs over a much longer time frame, such as several years.



Forecasting Models for Stationary Time Series

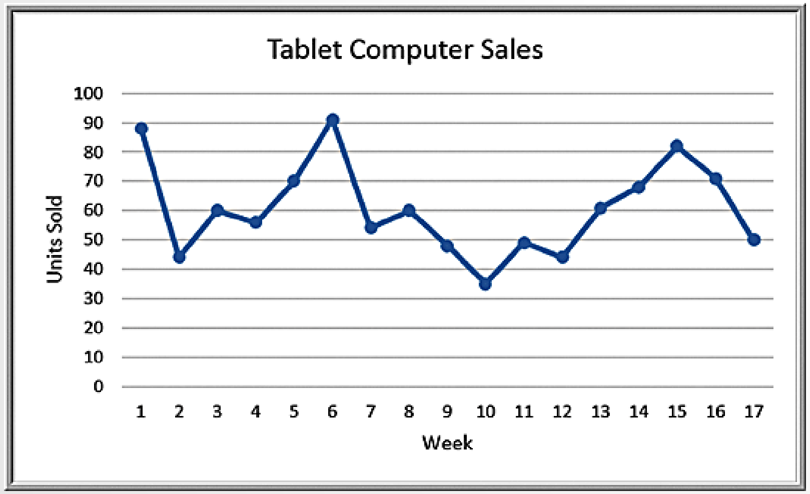
- Moving average model
- Exponential smoothing model
  - These are useful over short time periods when trend, seasonal, or cyclical effects are not significant

Moving Average Models

- The **simple moving average** method is a smoothing method based on the idea of averaging random fluctuations in the time series to identify the underlying direction in which the time series is changing.
- The simple moving average forecast for the next period is computed as the average of the most recent *k* observations.
  - Larger values of *k* result in smoother forecast models since extreme values have less impact.

Example 9.5: Moving Average Forecasting

- The *Tablet Computer Sales* data contains the number of units sold over the past 17 weeks.



- Three-period moving average forecast for week 18:

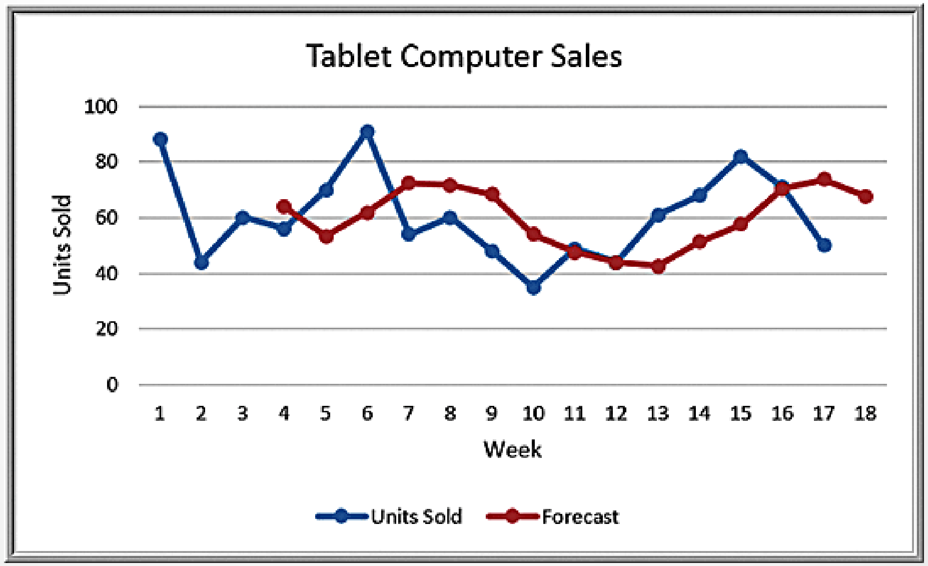
$$\text{week 18 forecast} = \frac{82 + 71 + 50}{3} = 67.67$$

Spreadsheet Implementation of Moving Average Forecasting

	A	B	C
1	**Tablet Computer Sales**		
2			**Moving Average**
3	**Week**	**Units Sold**	**Forecast**
4	1	88	
5	2	44	
6	3	60	
7	4	56	64.00
8	5	70	53.33
9	6	91	62.00
10	7	54	72.33
11	8	60	71.67
12	9	48	68.33
13	10	35	54.00
14	11	49	47.67
15	12	44	44.00
16	13	61	42.67
17	14	68	51.33
18	15	82	57.67
19	16	71	70.33
20	17	50	73.67
21	18		67.67
22			

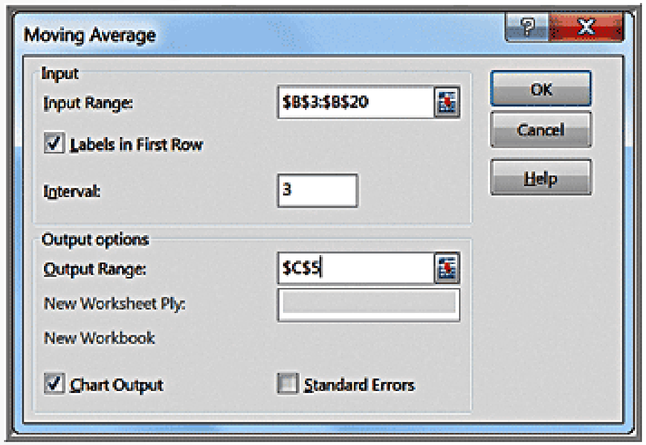
Forecast for week 4
=AVERAGE(B4:B6)

Forecast for week 18
=AVERAGE(B18:B20)

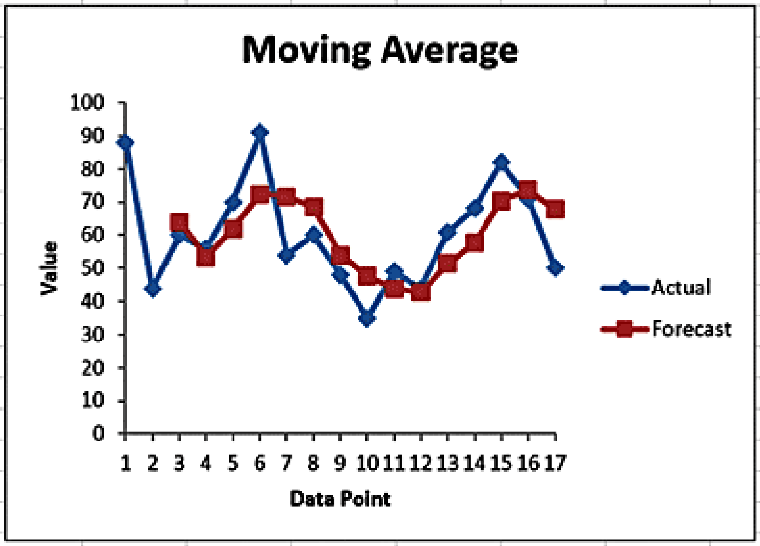


Excel *Moving Average* Tool

- *Data Analysis* options



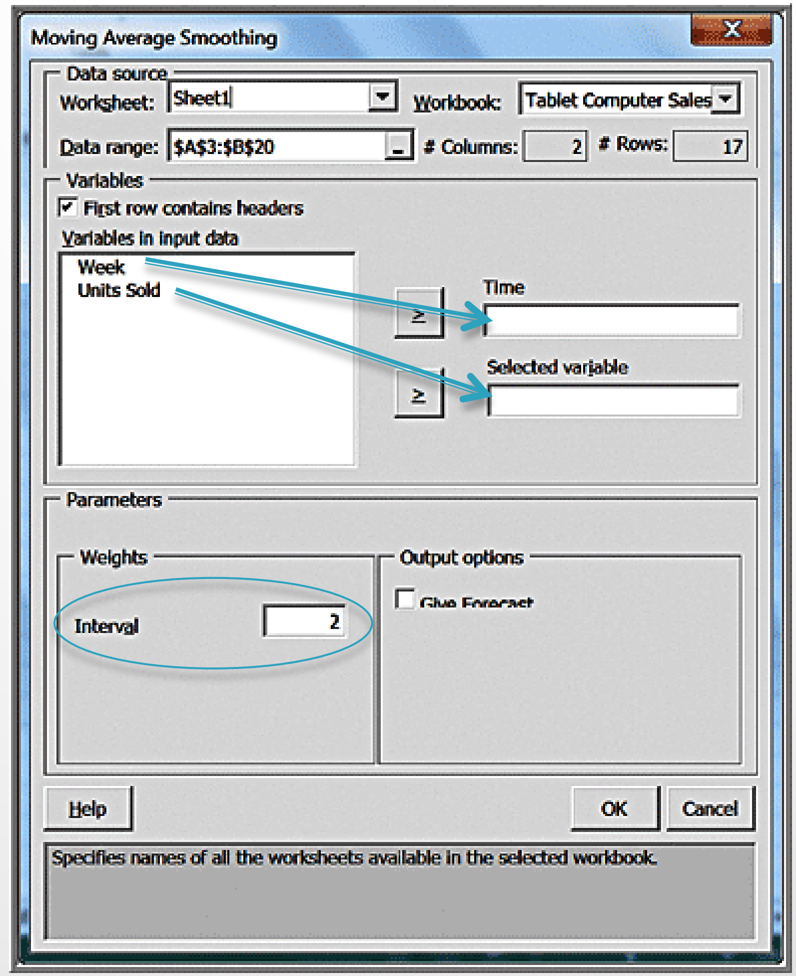
	A	B	C
1	Tablet Computer Sales		
2			Data Analysis
3	Week	Units Sold	MA Forecast
4	1	88	
5	2	44	#N/A
6	3	60	#N/A
7	4	56	64.00
8	5	70	53.33
9	6	91	62.00
10	7	54	72.33
11	8	60	71.67
12	9	48	68.33
13	10	35	54.00
14	11	49	47.67
15	12	44	44.00
16	13	61	42.67
17	14	68	51.33
18	15	82	57.67
19	16	71	70.33
20	17	50	73.67
21	18		67.67



We do not recommend using the chart or error options because the forecasts generated by this tool are not properly aligned with the data

Example 9.7: Moving Average Forecasting with *XLMiner*

- Select *Smoothing* from the *Time Series* group and select *Moving Average*
- Enter the data range and move the time variable and dependent variable to the boxes on the right. Enter the interval (*k*).



Examnle 9.7 Continued

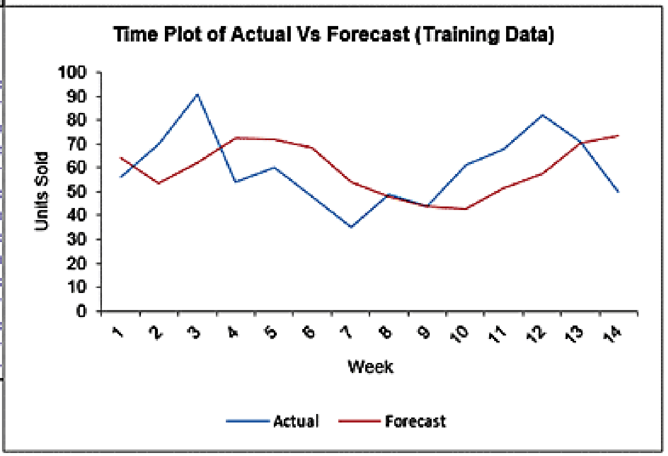
- *XLMiner* results

XLMiner : Time Series - Moving Average Smoothing

Output Navigator			
Inputs	Fitted Model	Forecast	
Elapsed Time	Error Measures(Training)		Error Measures(Validation)

Fitted Model

Week	Actual	Forecast	Residuals
1	88	*	*
2	44	*	*
3	60	*	*
4	56	64	-8
5	70	53.33333333	16.66666667
6	91	62	29
7	54	72.33333333	-18.33333333
8	60	71.66666667	-11.66666667
9	48	68.33333333	-20.33333333
10	35	54	-19
11	49	47.66666667	1.333333333
12	44	44	0
13	61	42.66666667	18.33333333
14	68	51.33333333	16.66666667
15	82	57.66666667	24.33333333
16	71	70.33333333	0.666666667
17	50	73.66666667	-23.66666667



Error Measures (Training)

MAPE	25.37413511
MAD	14.85714286
MSE	299.8412698

Forecast

Week	Forecast	LCI	UCI
Forecast1	67.66666667	36.86733215	98.46600118

Error Metrics and Forecast Accuarcy

- Mean absolute deviation (MAD)

$$\text{MAD} = \frac{\sum_{t=1}^{n} |A_t - F_t|}{n} \qquad (9.1)$$

- Mean square error (MSE)

$$\text{MSE} = \frac{\sum_{t=1}^{n} (A_t - F_t)^2}{n} \qquad (9.2)$$

- Root mean square error (RMSE)

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n} (A_t - F_t)^2}{n}} \qquad (9.3)$$

- Mean absolute percentage error (MAPE)

$$\text{MAPE} = \frac{\sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|}{n} \times 100 \qquad (9.4)$$

Example 9.8: Using Error Metrics to Compare Moving Average Forecasts

- *Tablet Computer Sales* data
- 2-, 3-, and 4-period moving average models
- 2-period model ahs lowest error metric values

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Tablet Computer Sales																
2			k = 2					k = 3					k = 4				
3	Week	Units Sold	Forecast	Error	Absolute	Squared	Absolute	Forecast	Error	Absolute	Squared	Absolute	Forecast	Error	Absolute	Squared	Absolute
4	1	88			Deviation	Error	% Error			Deviation	Error	% Error			Deviation	Error	% Error
5	2	44															
6	3	60	66.00	-6.00	6.00	36.00	10.00										
7	4	56	52.00	4.00	4.00	16.00	7.14	64.00	-8.00	8.00	64.00	14.29					
8	5	70	58.00	12.00	12.00	144.00	17.14	53.33	16.67	16.67	277.78	23.81	62.00	8.00	8.00	64.00	11.43
9	6	91	63.00	28.00	28.00	784.00	30.77	62.00	29.00	29.00	841.00	31.87	57.50	33.50	33.50	1122.25	36.81
10	7	54	80.50	-26.50	26.50	702.25	49.07	72.33	-18.33	18.33	336.11	33.95	69.25	-15.25	15.25	232.56	28.24
11	8	60	72.50	-12.50	12.50	156.25	20.83	71.67	-11.67	11.67	136.11	19.44	67.75	-7.75	7.75	60.06	12.92
12	9	48	57.00	-9.00	9.00	81.00	18.75	68.33	-20.33	20.33	413.44	42.36	68.75	-20.75	20.75	430.56	43.23
13	10	35	54.00	-19.00	19.00	361.00	54.29	54.00	-19.00	19.00	361.00	54.29	63.25	-28.25	28.25	798.06	80.71
14	11	49	41.50	7.50	7.50	56.25	15.31	47.67	1.33	1.33	1.78	2.72	49.25	-0.25	0.25	0.06	0.51
15	12	44	42.00	2.00	2.00	4.00	4.55	44.00	0.00	0.00	0.00	0.00	48.00	-4.00	4.00	16.00	9.09
16	13	61	46.50	14.50	14.50	210.25	23.77	42.67	18.33	18.33	336.11	30.05	44.00	17.00	17.00	289.00	27.87
17	14	68	52.50	15.50	15.50	240.25	22.79	51.33	16.67	16.67	277.78	24.51	47.25	20.75	20.75	430.56	30.51
18	15	82	64.50	17.50	17.50	306.25	21.34	57.67	24.33	24.33	592.11	29.67	55.50	26.50	26.50	702.25	32.32
19	16	71	75.00	-4.00	4.00	16.00	5.63	70.33	0.67	0.67	0.44	0.94	63.75	7.25	7.25	52.56	10.21
20	17	50	76.50	-26.50	26.50	702.25	53.00	73.67	-23.67	23.67	560.11	47.33	70.50	-20.50	20.50	420.25	41.00
21	18		60.50		13.63	254.38	23.63	67.67		14.86	299.84	25.37	67.75		16.13	355.25	28.07
22					MAD	MSE	MAPE			MAD	MSE	MAPE			MAD	MSE	MAPE

Exponential Smoothing Models

- **Simple exponential smoothing** model:

$$F_{t+1} = (1 - \alpha)F_t + \alpha A_t$$
$$= F_t + \alpha(A_t - F_t) \qquad \textbf{(9.5)}$$

where F_{t+1} is the forecast for time period $t + 1$, F_t is the forecast for period t, A_t is the observed value in period t, and α is a constant between 0 and 1 called the **smoothing constant**.

- To begin, set F_1 and F_2 equal to the actual observation in period 1, A_1.

Example 9.9: Using Exponential Smoothing to Forecast *Tablet Computer Sales*

	A	B	C	D	E	F	G	H	I	J	K
1	Tablet Computer Sales										
2							Smoothing Constant				
3	Week	Units Sold	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
4	1	88	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
5	2	44	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
6	3	60	83.60	79.20	74.80	70.40	66.00	61.60	57.20	52.80	48.40
7	4	56	81.24	75.36	70.36	66.24	63.00	60.64	59.16	58.56	58.84
8	5	70	78.72	71.49	66.05	62.14	59.50	57.86	56.95	56.51	56.28
9	6	91	77.84	71.19	67.24	65.29	64.75	65.14	66.08	67.30	68.63
10	7	54	79.16	75.15	74.37	75.57	77.88	80.66	83.53	86.26	88.76
11	8	60	76.64	70.92	68.26	66.94	65.94	64.66	62.86	60.45	57.48
12	9	48	74.98	68.74	65.78	64.17	62.97	61.87	60.86	60.09	59.75
13	10	35	72.28	64.59	60.45	57.70	55.48	53.55	51.86	50.42	49.17
14	11	49	68.55	58.67	52.81	48.62	45.24	42.42	40.06	38.08	36.42
15	12	44	66.60	56.74	51.67	48.77	47.12	46.37	46.32	46.82	47.74
16	13	61	64.34	54.19	49.37	46.86	45.56	44.95	44.70	44.56	44.37
17	14	68	64.00	55.55	52.86	52.52	53.28	54.58	56.11	57.71	59.34
18	15	82	64.40	58.04	57.40	58.71	60.64	62.63	64.43	65.94	67.13
19	16	71	66.16	62.83	64.78	68.03	71.32	74.25	76.73	78.79	80.51
20	17	50	66.65	64.47	66.65	69.22	71.16	72.30	72.72	72.56	71.95
21	18		64.98	61.57	61.65	61.53	60.58	58.92	56.82	54.51	52.20
22		MAD	19.33	17.16	16.15	15.36	14.93	14.71	14.72	14.88	15.36
23		MSE	496.07	390.84	359.18	346.56	340.77	338.41	339.03	343.32	352.38
24		MAPE	38.28%	32.71%	30.12%	28.36%	27.54%	27.09%	27.09%	27.38%	28.23%

Forecast for week 3 when $\alpha = 0.7$: $(1 – 0.7)(88) + (0.7)(44) = 57.2$

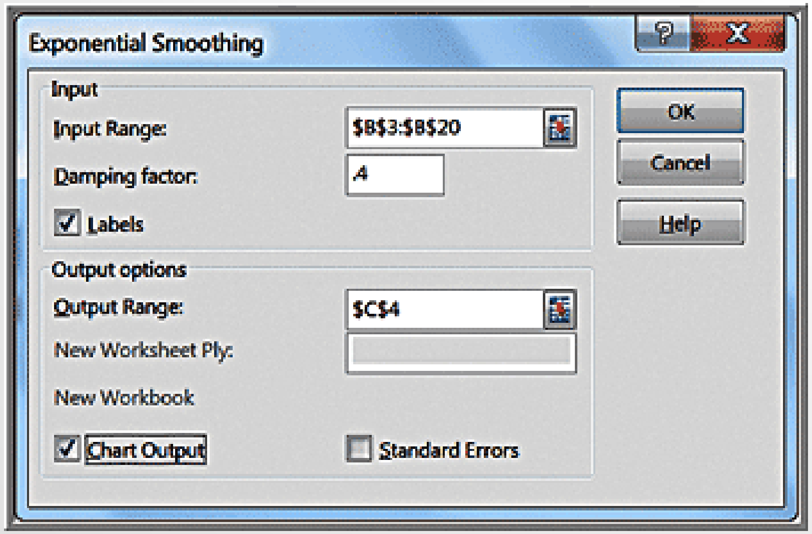
Example 9.10: Finding the Best Exponential Smoothing Model for *Tablet Computer Sales*

	A	B	C	D	E	F	G	H	I	J	K
1	Tablet Computer Sales										
2			Smoothing Constant								
3	Week	Units Sold	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.9
4	1	88	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.0
5	2	44	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.0
6	3	60	83.60	79.20	74.80	70.40	66.00	61.60	57.20	52.80	48.4
7	4	56	81.24	75.36	70.36	66.24	63.00	60.64	59.16	58.56	58.84
8	5	70	78.72	71.49	66.05	62.14	59.50	57.86	56.95	56.51	56.2
9	6	91	77.84	71.19	67.24	65.29	64.75	65.14	66.08	67.30	68.6
10	7	54	79.16	75.15	74.37	75.57	77.88	80.66	83.53	86.26	88.7
11	8	60	76.64	70.92	68.26	66.94	65.94	64.66	62.86	60.45	57.4
12	9	48	74.98	68.74	65.78	64.17	62.97	61.87	60.86	60.09	59.7
13	10	35	72.28	64.59	60.45	57.70	55.48	53.55	51.86	50.42	49.17
14	11	49	68.55	58.67	52.81	48.62	45.24	42.42	40.06	38.08	36.4
15	12	44	66.60	56.74	51.67	48.77	47.12	46.37	46.32	46.82	47.7
16	13	61	64.34	54.19	49.37	46.86	45.56	44.95	44.70	44.56	44.37
17	14	68	64.00	55.55	52.86	52.52	53.28	54.58	56.11	57.71	59.34
18	15	82	64.40	58.04	57.40	58.71	60.64	62.63	64.43	65.94	67.1
19	16	71	66.16	62.83	64.78	68.03	71.32	74.25	76.73	78.79	80.51
20	17	50	66.65	64.47	66.65	69.22	71.16	72.30	72.72	72.56	71.9
21	18		64.98	61.57	61.65	61.53	60.58	58.92	56.82	54.51	52.2
22		MAD	19.33	17.16	16.15	15.36	14.93	14.71	14.72	14.88	15.3
23		MSE	496.07	390.84	359.18	346.56	340.77	338.41	339.03	343.32	352.3
24		MAPE	38.28%	32.71%	30.12%	28.36%	27.54%	27.09%	27.09%	27.38%	28.23%

The forecast using $\alpha = 0.6$ provides the lowest error metrics.

Example 9.11: Using Excel's Exponential Smoothing Tool

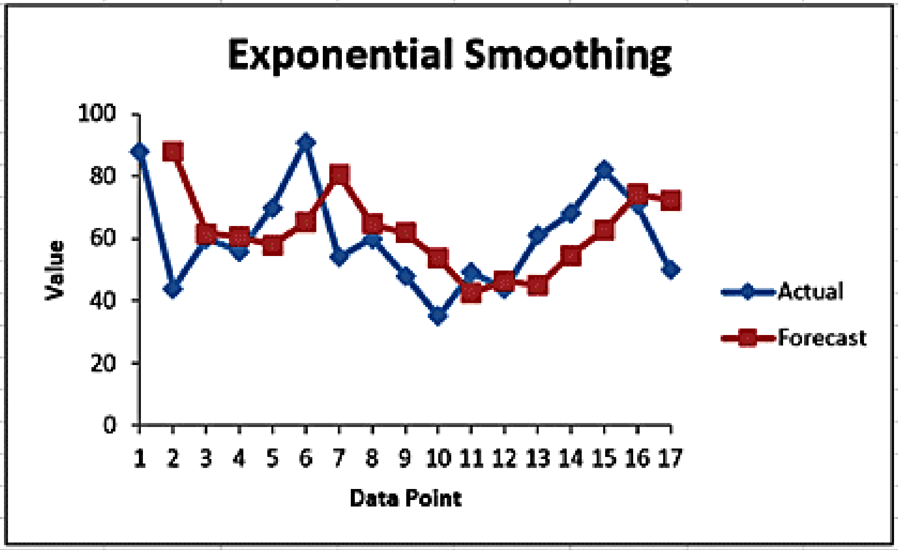
- Select *Data Analysis* from the *Analysis* group and then choose *Exponential Smoothing*
- Note that *Damping factor* = 1−α
- The first cell of the *Output Range* should be adjacent to the first data point



Example 9.11 Continued

- *Exponential Smoothing* tool results

	A	B	C
1	Tablet Computer Sales		
2			Exponential Smoothing
3	Week	Units Sold	Forecast
4	1	88	#N/A
5	2	44	88.00
6	3	60	61.60
7	4	56	60.64
8	5	70	57.86
9	6	91	65.14
10	7	54	80.66
11	8	60	64.66
12	9	48	61.87
13	10	35	53.55
14	11	49	42.42
15	12	44	46.37
16	13	61	44.95
17	14	68	54.58
18	15	82	62.63
19	16	71	74.25
20	17	50	72.30



Exponential Smoothing in *XLMiner*

- Select *Exponential* from the *Time Series/Smoothing* menu.
- Within the *Weights* pane, it provides options to either enter the smoothing constant, *Level (Alpha)* or to check an *Optimize* box, which will find the best value of the smoothing constant.

Example 9.12: Optimizing Exponential Smoothing Forecasts Using *XLMiner*

- The best smoothing constant is 0.63

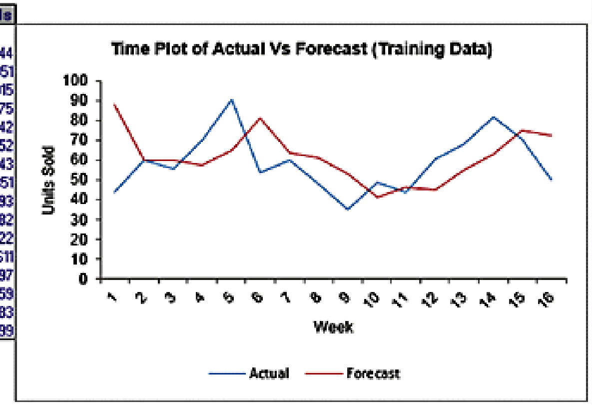
XLMiner : Time Series - Exponential Smoothing

Output Navigator		
Inputs	Fitted Model	Forecast
Elapsed Time	Error Measures(Training)	Error Measures(Validation)

Parameters/Options	
Optimization Selected	Yes
Alpha (Level)	0.630925524
Beta (Trend)	N.A.
Gamma (Seasonality)	N.A.
Season length	N.A.
Number of seasons	N.A.
Forecast	Yes
#Forecasts	1

Fitted Model

Week	Actual	Forecast	Residuals
1	88	*	*
2	44	88	-44
3	60	60.23927695	-0.239276951
4	56	60.08831102	-4.088311015
5	70	57.50889125	12.49110875
6	91	65.38985058	25.61014942
7	54	81.54794752	-27.54794752
8	60	64.1672443	-4.1672443
9	48	61.53802351	-13.53802351
10	35	52.99653893	-17.99653893
11	49	41.64206318	7.35793682
12	44	46.28437332	-2.284373322
13	61	44.84310389	16.15689611
14	68	55.03690203	12.96309797
15	82	63.21565141	18.78434859
16	71	75.06717638	-4.067176383
17	50	72.50109099	-22.50109099



Error Measures (Training)

MAPE	26.91663918
MAD	14.61209504
MSE	338.2640932

Forecast

Week	Forecast	LCI	UCI
Forecast1	58.30457837	23.33265228	93.27650446

Forecasting Models for Time Series with a Linear Trend

- **Double moving average** and **double exponential smoothing**
- Based on the linear trend equation

$$F_{t+k} = a_t + b_t k \qquad (9.6)$$

- The forecast for *k* periods into the future is a function of the *level* a_t and the trend b_t
- The models differ in their computations of a_t and b_t
- *XLMiner* does not support a procedure for double moving average; however, it does provide one for double exponential smoothing.

Double Exponential Smoothing

- Estimates of the parameters are obtained from the following equations:

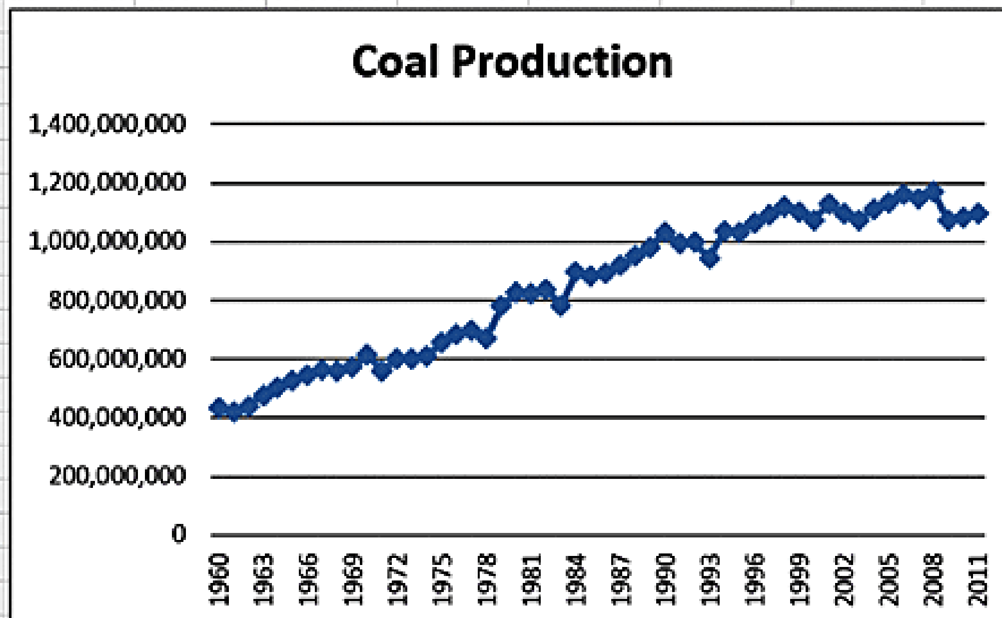
$$a_t = \alpha F_t + (1 - \alpha)(a_{t-1} + b_{t-1})$$
$$b_t = \beta(a_t - a_{t-1}) + (1 - \beta)b_{t-1} \qquad (9.7)$$

- Initial values are chosen for a_1 as A_1 and b_1 as $A_2 - A_1$. Equation (9.7) must then be used to compute a_t and b_t for the entire time series to be able to generate forecasts into the future.

Example 9.13: Double Exponential Smoothing with *XLMiner*

- Excel file *Coal Production*

	A	B
1	Coal Production	
2		
3	Year	Total Tons
4	1960	434,329,000
5	1961	420,423,000
6	1962	439,043,000
7	1963	477,195,000
8	1964	504,182,000
9	1965	526,954,000
10	1966	546,822,000
11	1967	564,882,000
12	1968	556,706,000
13	1969	570,978,000
14	1970	612,661,000
15	1971	560,919,000
16	1972	602,492,000
17	1973	598,568,000
18	1974	610,023,000
19	1975	654,641,000



Example 9.13 Continued

- *XLMiner* optimization results
- the best values of α and β are 0.684 and 0.00

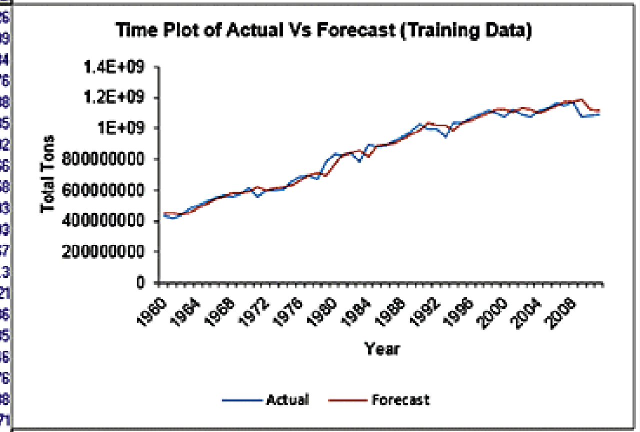
XLMiner : Time Series - Double Exponential Smoothing

Output Navigator		
Inputs	Fitted Model	Forecast
Elapsed Time	Error Measures(Training)	Error Measures(Validation)

Parameters/Options	
Optimization Selected	Yes
Alpha (Level)	0.684530657
Beta (Trend)	1.91197E-15
Gamma (Seasonality)	N.A.
Season length	N.A.
Number of seasons	N.A.
Forecast	Yes
#Forecasts	3

Fitted Model

Year	Actual	Forecast	Residuals
1960	434329000	454233422.3	-19904422.26
1961	420423000	456021772	-35598771.99
1962	439043000	447066858.2	-8023858.184
1963	477195000	456987818.2	20207181.76
1964	504182000	486233790.6	17948209.38
1965	526954000	513933427.1	13020572.85
1966	546822000	539259945.4	8562054.592
1967	564882000	559534471.2	5347528.766
1968	556706000	578608555.6	-21902555.58
1969	570978000	579029121.8	-8051121.793
1970	612661000	588931419.1	23729580.93
1971	560919000	620588581.7	-59669581.67
1972	602492000	595156460.7	7335539.3
1973	598568000	615591399.2	-17023399.21
1974	610023000	619351897.5	-9328897.536
1975	654641000	628379518.1	26261481.85
1976	684913000	661769844.5	23143155.46
1977	697205000	693025580.9	4179419.076
1978	670164000	711300058.4	-41136058.38
1979	781134000	698554702.3	82579297.71
1980	829700000	770496300.2	59203699.83

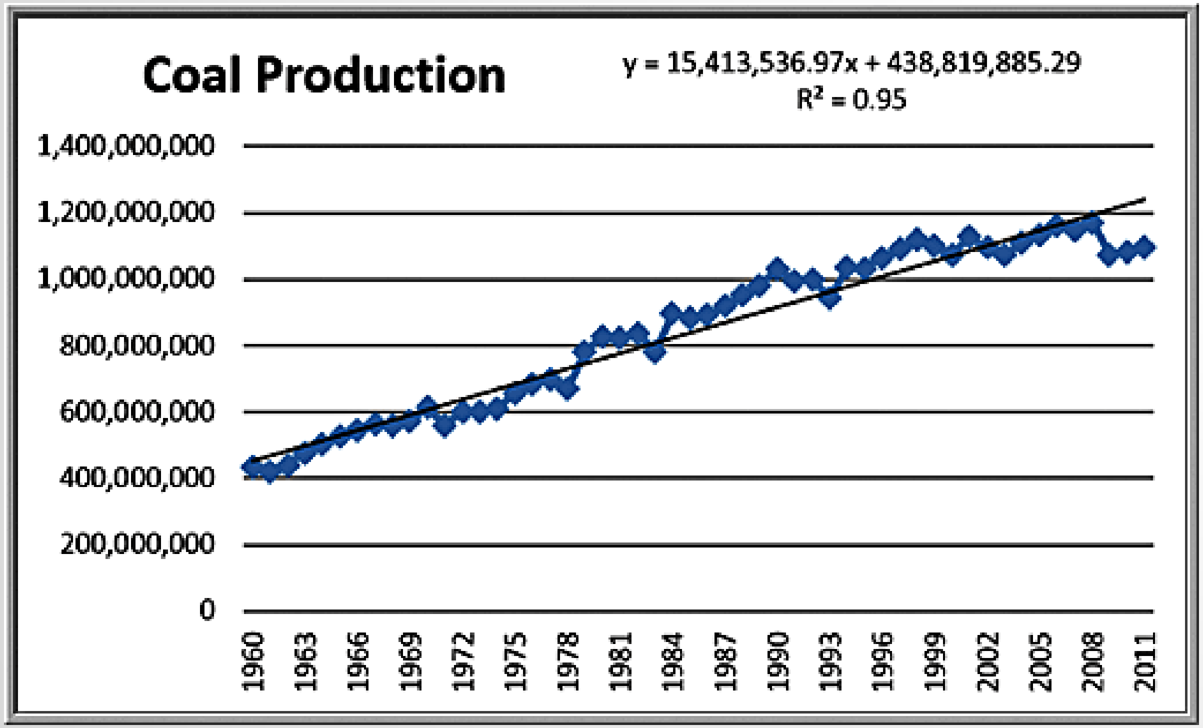


Regression-Based Forecasting for Time Series with a Linear Trend

- Simple linear regression can be applied to forecasting using time as the independent variable.

Example 9.14: Forecasting Using Trendlines

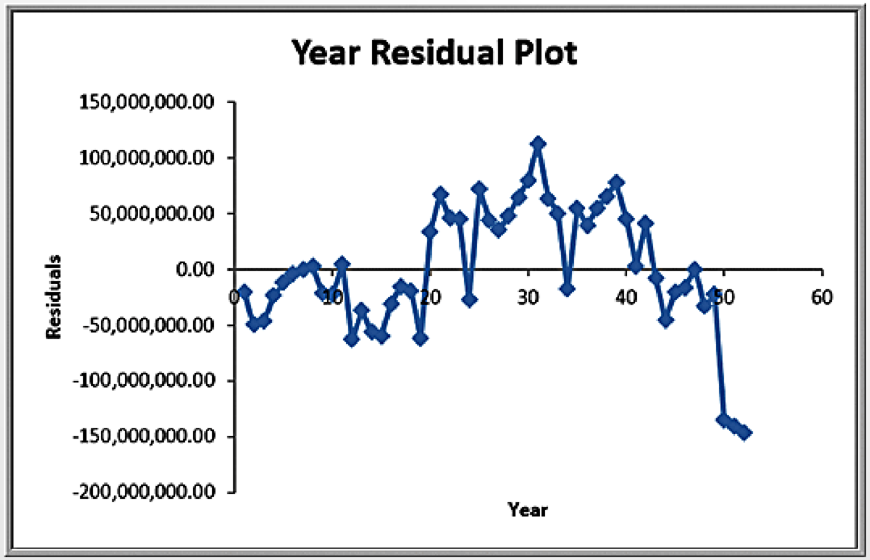
- *Coal Production* data with a linear trendline



Note that the linear model does not adequately predict the recent drop in production after 2008.

Autocorrelation in Time Series

- When autocorrelation is present, successive observations are correlated with one another; for example, large observations tend to follow other large observations, and small observations also tend to follow one another.
  - In such cases, other approaches, called **autoregressive models**, are more appropriate.

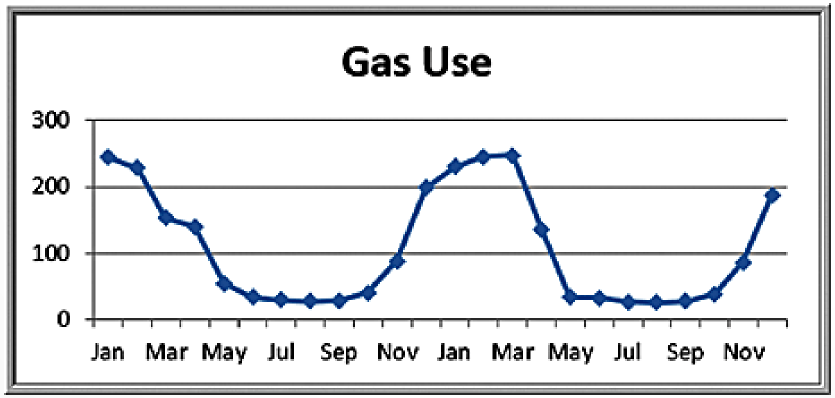


Forecasting Time Series with Seasonality

- When time series exhibit seasonality, different techniques provide better forecasts than the ones we have described:
  - Multiple regression models with categorical variables for the seasonal components
  - Holt-Winters models, similar to exponential smoothing models in that smoothing constants are used to smooth out variations in the level and seasonal patterns over time. For time series with seasonality but no trend, *XLMiner* supports a Holt-Winters method but does not have the ability to optimize the parameters.

Example 9.15: Regression-Based Forecasting for Natural Gas Usage

- *Gas & Electric* Excel file
- Use a seasonal categorical
- variable with k = 12 levels.
- Construct the regression model using k - 1 dummy variables, with January being the reference month.



$$\begin{aligned}\text{gas usage} = {} & \beta_0 + \beta_1 \text{ time} + \beta_2 \text{ February} + \beta_3 \text{ March} \\ & + \beta_4 \text{ April} + \beta_5 \text{ May} + \beta_6 \text{ June} + \beta_7 \text{ July} \\ & + \beta_8 \text{ August} + \beta_9 \text{ September} + \beta_{10} \text{ October} \\ & + \beta_{11} \text{ November} + \beta_{12} \text{ December}\end{aligned}$$

Example 9.15 Continued

- Data matrix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Gas and Electric Usage**													
2														
3	**Month**	**Gas Use**	**Time**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
4	Jan	244	1	0	0	0	0	0	0	0	0	0	0	0
5	Feb	228	2	1	0	0	0	0	0	0	0	0	0	0
6	Mar	153	3	0	1	0	0	0	0	0	0	0	0	0
7	Apr	140	4	0	0	1	0	0	0	0	0	0	0	0
8	May	55	5	0	0	0	1	0	0	0	0	0	0	0
9	Jun	34	6	0	0	0	0	1	0	0	0	0	0	0
10	Jul	30	7	0	0	0	0	0	1	0	0	0	0	0
11	Aug	28	8	0	0	0	0	0	0	1	0	0	0	0
12	Sep	29	9	0	0	0	0	0	0	0	1	0	0	0
13	Oct	41	10	0	0	0	0	0	0	0	0	1	0	0
14	Nov	88	11	0	0	0	0	0	0	0	0	0	1	0
15	Dec	199	12	0	0	0	0	0	0	0	0	0	0	1
16	Jan	230	13	0	0	0	0	0	0	0	0	0	0	0
17	Feb	245	14	1	0	0	0	0	0	0	0	0	0	0
18	Mar	247	15	0	1	0	0	0	0	0	0	0	0	0
19	Apr	135	16	0	0	1	0	0	0	0	0	0	0	0
20	May	34	17	0	0	0	1	0	0	0	0	0	0	0
21	Jun	33	18	0	0	0	0	1	0	0	0	0	0	0
22	Jul	27	19	0	0	0	0	0	1	0	0	0	0	0
23	Aug	26	20	0	0	0	0	0	0	1	0	0	0	0
24	Sep	28	21	0	0	0	0	0	0	0	1	0	0	0
25	Oct	39	22	0	0	0	0	0	0	0	0	1	0	0
26	Nov	86	23	0	0	0	0	0	0	0	0	0	1	0
27	Dec	188	24	0	0	0	0	0	0	0	0	0	0	1

Example 9.15 Continued

- Final regression results (time and February were insignificant)

	A	B	C	D	E	F	G	H	I
1	SUMMARY OUTPUT								
2									
3	*Regression Statistics*								
4	Multiple R	0.985480895							
5	R Square	0.971172595							
6	Adjusted R Square	0.948997667							
7	Standard Error	19.54432831							
8	Observations	24							
9									
10	ANOVA								
11		*df*	*SS*	*MS*	*F*	*Significance F*			
12	Regression	10	167292.2083	16729.22083	43.79597661	2.33344E-08			
13	Residual	13	4965.75	381.9807692					
14	Total	23	172257.9583						
15									
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
17	Intercept	236.75	9.772164157	24.22697738	3.33921E-12	215.6385228	257.8614772	215.6385228	257.8614772
18	Mar	-36.75	16.92588482	-2.171230656	0.049016211	-73.31615105	-0.183848953	-73.31615105	-0.183848953
19	Apr	-99.25	16.92588482	-5.863799799	5.55744E-05	-135.816151	-62.68384895	-135.816151	-62.68384895
20	May	-192.25	16.92588482	-11.35834268	4.02824E-08	-228.816151	-155.683849	-228.816151	-155.683849
21	Jun	-203.25	16.92588482	-12.00823485	2.07264E-08	-239.816151	-166.683849	-239.816151	-166.683849
22	Jul	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849	-244.816151	-171.683849
23	Aug	-209.75	16.92588482	-12.39226204	1.41949E-08	-246.316151	-173.183849	-246.316151	-173.183849
24	Sep	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849	-244.816151	-171.683849
25	Oct	-196.75	16.92588482	-11.62420766	3.05791E-08	-233.316151	-160.183849	-233.316151	-160.183849
26	Nov	-149.75	16.92588482	-8.847395666	7.30451E-07	-186.316151	-113.183849	-186.316151	-113.183849
27	Dec	-43.25	16.92588482	-2.555257847	0.023953114	-79.81615105	-6.683848953	-79.81615105	-6.683848953

$$
\begin{aligned}
\text{gas usage} = {} & 236.75 - 36.75\ \text{March} - 99.25\ \text{April} \\
& - 192.25\ \text{May} - 203.25\ \text{June} - 208.25\ \text{July} \\
& - 209.75\ \text{August} - 208.25\ \text{September} \\
& - 196.75\ \text{October} - 149.75\ \text{November} \\
& - 43.25\ \text{December}
\end{aligned}
$$

Holt-Winters Models for Forecasting Time Series with Seasonality and Trend

- The **Holt-Winters additive model** applies to time series with relatively stable seasonality and is based on the equation

$$F_{t+1} = a_t + b_t + S_{t-s+1} \qquad (9.8)$$

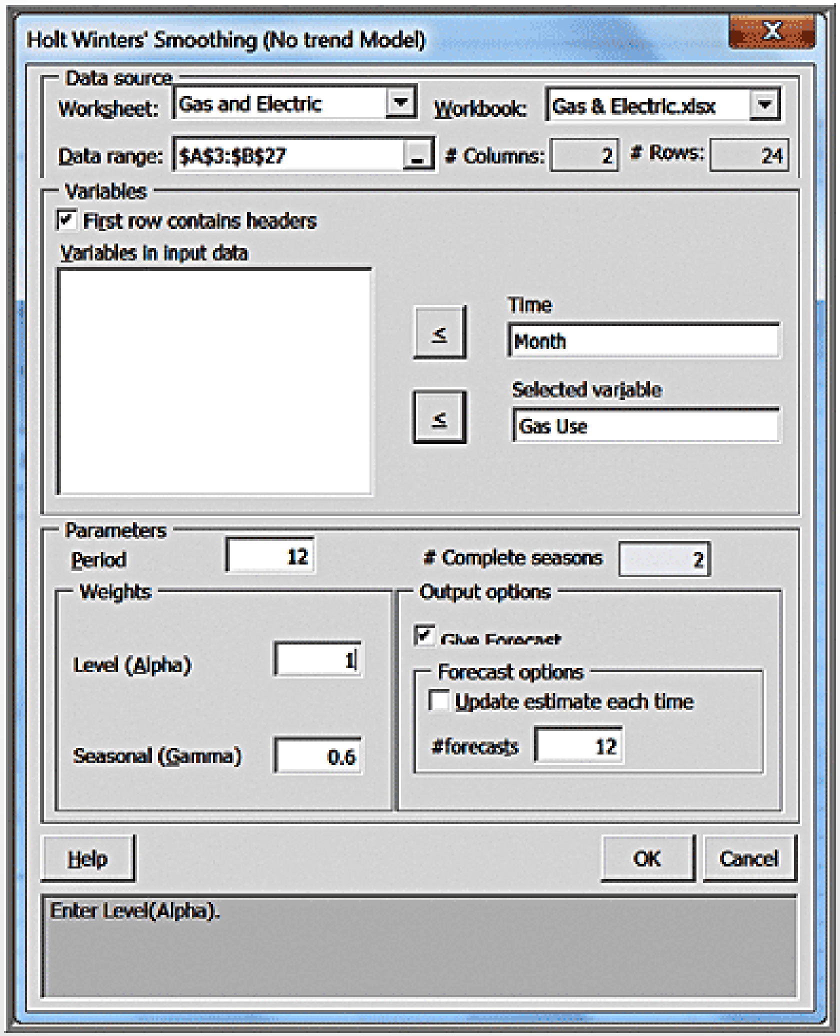
- The **Holt-Winters multiplicative model** applies to time series whose amplitude increases or decreases over time and is

$$F_{t+1} = (a_t + b_t)S_{t-s+1} \qquad (9.9)$$

- A chart of the time series should be viewed first to identify the appropriate type of model to use.

Example 9.16: Forecasting Natural Gas Usage Using Holt-Winters No-Trend Model

- *XLMiner > Time Series > Smoothing >Holt Winter No Trend…*
- In the *Parameters* pane, the value of *Period* is the length of the season, in this case, 12 months.
- You will generally have to experiment with the smoothing constants to identify the best model.

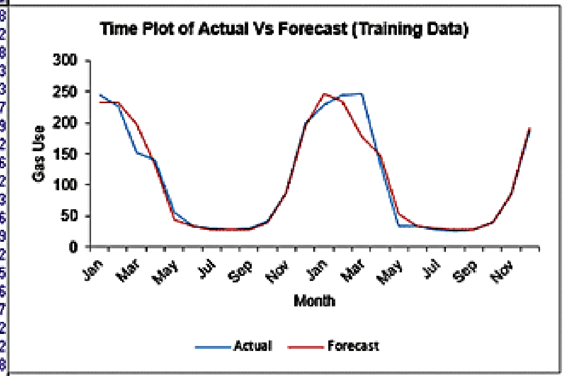


Example 9.16 Continued

- *XLMiner* results

Fitted Model

Month	Actual	Forecast	Residuals
Jan	244	232.5110166	11.48898338
Feb	228	233.1669616	-5.166961552
Mar	153	196.7444105	-43.74441048
Apr	140	132.254354	7.745646013
May	55	43.05299547	11.94700453
Jun	34	33.31006322	0.689936777
Jul	30	28.39710811	1.60289189
Aug	28	27.05437639	0.945623612
Sep	29	28.65721312	0.342786876
Oct	41	40.26876044	0.731239562
Nov	88	87.74359856	0.256401443
Dec	199	195.210893	3.789107026
Jan	230	245.9203019	-15.92030189
Feb	245	234.6582226	10.34177742
Mar	247	176.9643178	70.03568225
Apr	135	148.7161242	-13.71612416
May	34	52.97317647	-18.97317647
Jun	33	33.93217254	-0.932172542
Jul	27	29.3335162	-2.333516202
Aug	26	27.26084293	-1.260842928
Sep	28	28.29484269	-0.294842686
Oct	39	39.80219176	-0.802191759
Nov	86	85.69281212	0.307187885
Dec	188	192.3520753	-4.352075296



Error Measures (Training)

MAPE	8.595445716
MAD	9.488370193
MSE	338.9665158

Forecast

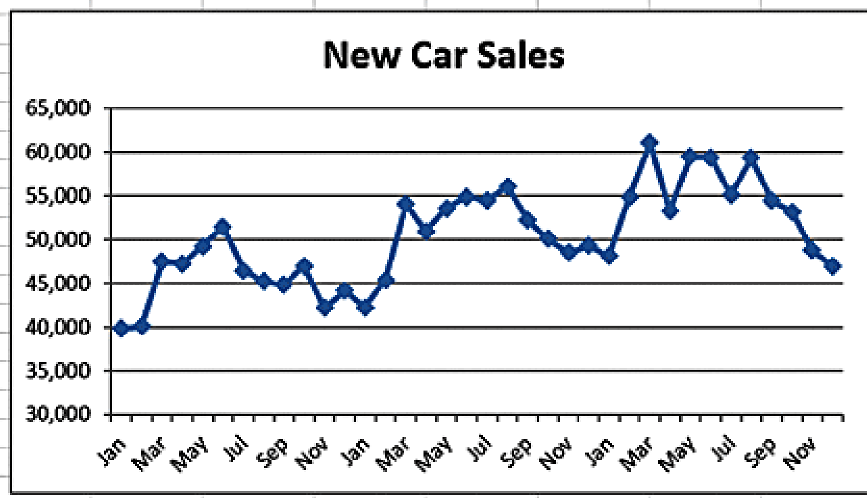
Month	Forecast	LCI	UCI
Forecast01	230.4014413	194.3157964	266.4870862
Forecast02	234.7869609	198.701316	270.8726058
Forecast03	207.6051864	171.5195415	243.6908313
Forecast04	132.208982	96.12333706	168.2946268
Forecast05	40.00822428	3.922579391	76.09386917
Forecast06	32.75435153	-3.331293362	68.83999642
Forecast07	27.57400817	-8.51163672	63.65965306
Forecast08	26.32283878	-9.762806111	62.40848367
Forecast09	27.99606567	-8.089579215	64.08171056
Forecast010	39.21394812	3.128303229	75.29959301
Forecast011	85.69508267	49.60943778	121.7807276
Forecast012	189.5667471	153.4811022	225.652392

Example 9.17: Forecasting *New Car Sales* Using Holt-Winters Models

- There is clearly a stable seasonal factor in the time series, along with an increasing trend; therefore, the Holt-Winters additive model would appear

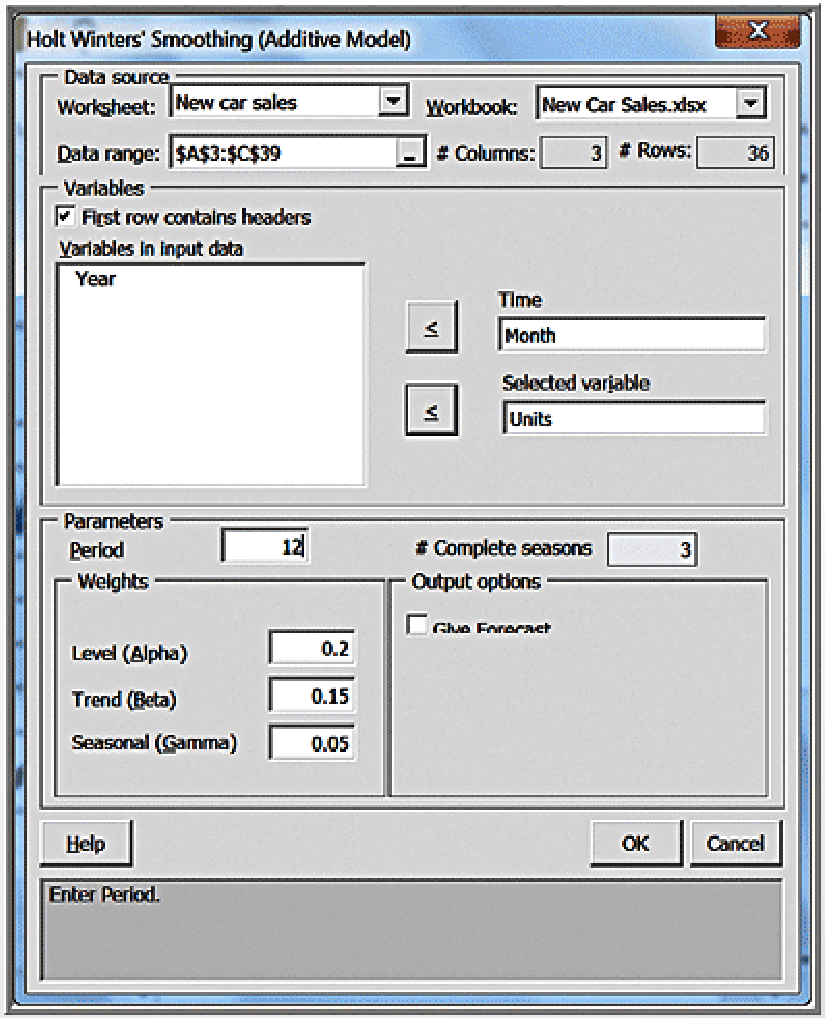
New Car Retail Sales

Year	Month	Units
1	Jan	39,810
1	Feb	40,081
1	Mar	47,440
1	Apr	47,297
1	May	49,211
1	Jun	51,479
1	Jul	46,466
1	Aug	45,208
1	Sep	44,800
1	Oct	46,989
1	Nov	42,161
1	Dec	44,186
2	Jan	42,227
2	Feb	45,422
2	Mar	54,075
2	Apr	50,926
2	May	53,572



Example 9.17: Continued

- *XLMiner > Time Series > Smoothing > Holt Winter Additive…*



Example 9.17 Continued

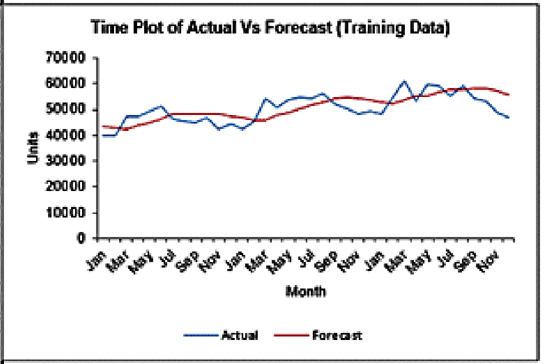
- *XLMiner* results

XLMiner : Time Series - Holt Winter Forecasting Method(Additive Model)

Output Navigator		
Inputs	Fitted Model	Forecast
Elapsed Time	Error Measures(Training)	Error Measures(Validation)

Fitted Model

Month	Actual	Forecast	Residuals
Jan	39810	43536.46201	-3726.46201
Feb	40081	43057.78914	-2976.78914
Mar	47440	42639.83285	4800.167148
Apr	47297	43921.04942	3375.950582
May	49211	45018.84132	4192.158683
Jun	51479	46405.59467	5073.405325
Jul	46466	48120.70874	-1654.70874
Aug	45208	48440.65231	-3232.65231
Sep	44800	48347.94308	-3547.94308
Oct	46989	48085.78966	-1096.78966
Nov	42161	48280.8989	-6119.8989
Dec	44186	47287.86812	-3101.86812
Jan	42227	46656.33112	-4429.33112
Feb	45422	45805.52236	-383.522355
Mar	54075	46033.5467	8041.453297
Apr	50926	47819.53944	3106.460556
May	53572	48801.48457	4770.515431
Jun	54920	50261.91275	4658.087248
Jul	54449	51535.13271	2913.867292
Aug	56079	52753.02509	3325.974907
Sep	52177	54203.53984	-2026.53984
Oct	50087	54633.46547	-4546.46547
Nov	48513	54123.97716	-5610.97716
Dec	49278	53554.9816	-4276.9816
Jan	48134	52801.59962	-4667.59962
Feb	54887	52224.09769	2662.902312
Mar	61064	53648.92692	7415.073084
Apr	53350	55343.97342	-1993.97342
May	59467	55451.11044	4015.88956
Jun	59370	56812.19404	2557.805959
Jul	55088	57588.65822	-2500.65822
Aug	59349	57570.76332	1778.236677
Sep	54472	58281.8312	-3809.8312
Oct	53164	57985.00416	-4821.00416
Nov	48793	57100.49411	-8307.49411
Dec	46956	55687.12594	-8731.12594



Error Measures (Training)

MAPE	8.050205544
MAD	4006.960117
MSE	19725609.01

Selecting Appropriate Time-Series-Based Forecasting Methods

	No Seasonality	Seasonality
No trend	Simple moving average or simple exponential smoothing	Holt-Winters no-trend smoothing model or multiple regression
Trend	Double exponential smoothing	Holt-Winters additive or Holt-Winters multiplicative model

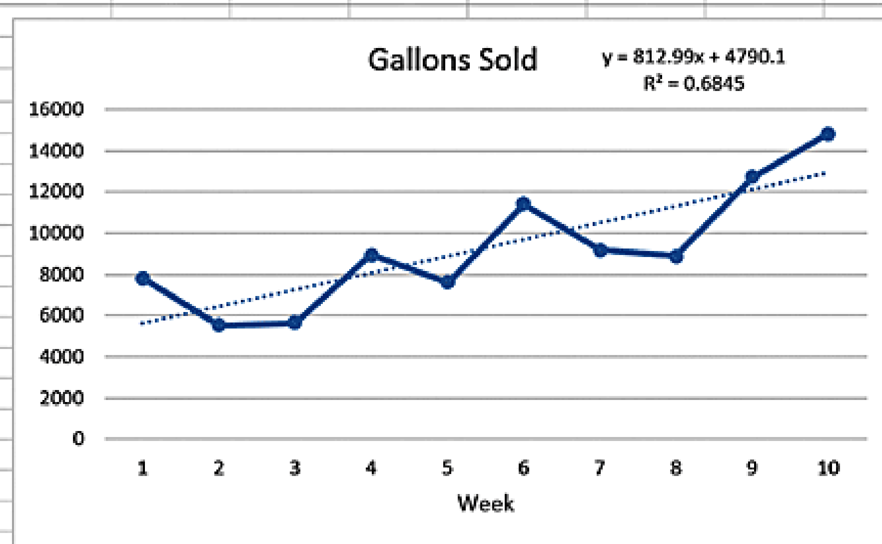
Regression Forecasting with Causal Variables

- In many forecasting applications, other independent variables besides time, such as economic indexes or demographic factors, may influence the time series.
- Explanatory/causal models, often called **econometric models**, seek to identify factors that explain statistically the patterns observed in the variable being forecast, usually with regression analysis

Example 9.18: Forecasting Gasoline Sales Using Simple Linear Regression

- Excel file *Gasoline Sales*
- Simple trendline using week as the independent variable

	A	B	C
1	**Gasoline Sales**		
2			
3	**Gallons Sold**	**Week**	**Price/Gallon**
4	7815	1	$3.95
5	5541	2	$4.20
6	5650	3	$4.12
7	8949	4	$3.98
8	7600	5	$4.01
9	11430	6	$3.92
10	9190	7	$4.03
11	8889	8	$3.98
12	12721	9	$3.92
13	14830	10	$3.90
14		11	$3.80



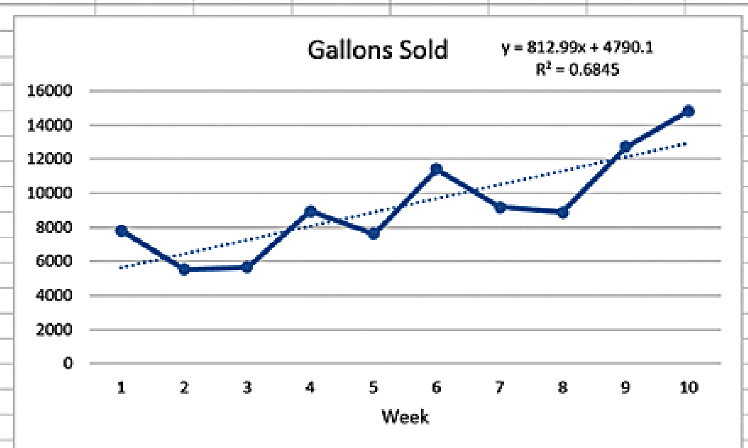
Predicted sales for week 11 = 812.99(11) + 4790.1 = 13,733 gallons

Example 9.19: Incorporating Causal Variables in a Regression-Based Forecasting Model

- The average price per gallon changes each week, and this may influence consumer sales. Average price per gallon is a *causal variable.*
- Develop a multiple linear regression model to predict gasoline sales using both time and price per gallon.

Gasoline Sales

Gallons Sold	Week	Price/Gallon
7815	1	$3.95
5541	2	$4.20
5650	3	$4.12
8949	4	$3.98
7600	5	$4.01
11430	6	$3.92
9190	7	$4.03
8889	8	$3.98
12721	9	$3.92
14830	10	$3.90
	11	$3.80



Example 9.19 Continued

- Multiple regression model $\text{sales} = \beta_0 + \beta_1 \text{ week} + \beta_2 \text{ price/gallon}$

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.930528528					
5	R Square	0.865883342					
6	Adjusted R Square	0.827564297					
7	Standard Error	1235.400329					
8	Observations	10					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	2	68974748.7	34487374.35	22.59668368	0.000883465	
13	Residual	7	10683497.8	1526213.972			
14	Total	9	79658246.5				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	72333.08447	21969.92267	3.292368642	0.013259225	20382.47252	124283.6964
18	Week	508.6681395	168.1770861	3.024598364	0.019260863	110.9925232	906.3437559
19	Price/Gallon	-16463.19901	5351.082403	-3.076611005	0.017900405	-29116.49823	-3809.899786

Predicted sales for week 11
= 72,333 + 508.7(11) – 16,463(3.80) = 15,368 gallons

The Practice of Forecasting

- Judgmental and qualitative methods are used for forecasting sales of product lines and broad company and industry forecasts.
- Simple time-series models are used for short- and medium-range forecasts.
- Regression methods are typically used for long-term forecasts.

Google Analytics GROUP Project 5 Email me (albert.kalim@asbury.edu) your GROUP answers by Friday, 7/8,11:59 p.m. ET (10 points total)

Log in to your Google Analytics dashboard here and click Sign in to Analytics.

After you logged in, focus on the following three tools on the left-hand side: **Audience, Acquisition, and Behavior**. Work with your group and **list three business recommendations for each of the tool and elaborate on them**. ***These recommendations must be new or different than your individual project 3 and 4 answers.*** You will be graded as a group and individually based on your familiarity with the tools and how you read/interpret the data.